2) $G_p(s)$, $G_r(s)$ are stable, G_p has no zeros at the origin, and $G_r(s)$, $[G_p(s)/s]$ are PR.

3) $\mathrm{Re}[G_r(j\omega)] > 0$ for all real ω.

Proof. From assumption 2, $C(s)$ is PR and has no unstable cancellations. From assumptions 1 and 2, $P(s)C(s)$ has no unstable cancellations. From assumption 3, $\mathrm{Re}[C(j\omega)]>0$ for all real ω. Therefore, from Theorem 1, A_{CL} is Hurwitz.

A special case of particular interest is the flexible spacecraft control problem, where P is given by Eq. (2) and G_p, G_r are positive constants, and a finite-bandwidth actuator, represented by a stable, proper, minimum-phase transfer function $G_a(s)$, is used in the loop. For this case,

$$C(s) = \frac{G_a(s)[G_p + sG_r]}{s} \qquad (16)$$

From Eq. (2), $P(s)$ is PR and has no zeros at $s = 0$. Therfore, from Theorem 1, A_{CL} is stable if $\mathrm{Re}[C(j\omega)]>0$ for all real ω, which will be satisfied if

$$-90 \text{ deg} < \phi[C(j\omega)] < 90 \text{ deg} \qquad \text{for } 0 \le \omega < \infty$$

where $\phi[.]$ denotes the phase angle of a complex variable. The preceding expression can be shown to be equivalent to

$$-\arctan(\omega G_r/G_p) < \phi[G_a(j\omega)] < 180 \text{ deg} - \arctan(\omega G_r/G_p)$$

$$\text{for } 0 \le \omega < \infty \qquad (17)$$

For the special case where $G_a(s) = k/(s+a)$(with $k > 0$, $a > 0$), Eq. (17) is equivalent to

$$a > G_p/G_r$$

That is, if the actuator bandwidth is greater than G_p/G_r, the closed-loop stability is guaranteed regardless of the number of modes in the model or parameter uncertainties. This result was proved in Ref. 4 for the multi-input/multi-output case for systems with constant, positive definite proportional and rate gain matrices, using function space methods.

Conclusions

A generalized proportional-plus-derivative compensator was proposed for robustly stabilizing a class of uncertain plants represented by a positive-real transfer function followed by an integrator. Such plants are encountered, for example, in the study of attitude control of large flexible space structures, wherein significant uncertainty exists in the model order as well as the parameter values. The novel feature of the proposed compensator is that the proportional and rate gains used are transfer functions rather than constants, which allows more design freedom, and offers the potential for obtaining better performance with guaranteed robust stability. The results obtained herein are for single-input/single-output systems. It would be highly desirable to extend the results to the multivariable case, which will then be applicable to realistic flexible spacecraft, and also to develop synthesis methods for such controllers.

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Order-Variable Adaptive Pole-Placement Controllers for a Flexible System

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I. Introduction

ORDER determination is an important problem in adaptive control of flexible systems because the number of plant modes excited varies with external excitation, speed of maneuvers, etc. Also, the number of excited modes can increase abruptly due to impulsive disturbances such as collisions or release of payload. The number of additional modes and their contribution to output often are larger than levels accommodated by robust controllers of lower order. In such applications, a major difficulty is to identify the correct order of the system (and adjust the control law). Here, we propose a natural approach to the issue of unknown, or varying, system order; change the order of the control law, as the need arises. Such a technique requires the ability to identify adaptively the appropriate order for the system while providing the parameter estimates corresponding to that order. Since this order is in general unknown, common identification algorithms, which have a fixed order, are not suitable for this purpose. The identification method used here is the lattice filter, which is an order-recursive method. Because of this order-recursive property, lattices provide the ability to obtain parameter estimates for any order model. The example in this paper involves a linear lumped-mass plant that increases in dimension after a collision during operation. Before the collision, the adaptive controller identifies a plant order and a corresponding set of parameters; after the collision, the controller must adjust both the order and the parameters. Simulations for the example show that the variable-order adaptive controller is much better than fixed-order controllers using the same control law. The adaptive control law used in this paper is an indirect pole-placement algorithm. Results presented show the advantages gained by allowing the order to change when the effective order of the system is either unknown or can change during operation.

II. Adaptive Control Law

For the example in Sec. IV indirect pole placement is chosen since it does not require the system to be minimum phase (e.g., see Ref. 1 for details). An auto regressive moving average (ARMA) model of the plant is identified adaptively, and the control gains are computed so that the closed-loop systems will have specified poles. Consider the input/output model

$$A(q^{-1})y(k) = B(q^{-1})u(k) \qquad (1)$$

where $A(q^{-1}) = 1 - a_1q^{-1} - \cdots - a_nq^{-n}$, and q^{-1} is the backward shift; i.e., $q^{-1}y(k) = y(k-1)$. The control has the following form (see Ref. 2 for details of the pole-placement algorithm):

$$L(q^{-1})u(k) = P(q^{-1})[y^*(k+1) - y(k)] \qquad (2)$$

where $L(q^{-1})$ and $P(q^{-1})$ are $(n-1)$ order polynomials in q^{-1}. Using Eq. (2) in Eq. (1) results in the following equation

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for the closed-loop system:

$$[L(q^{-1})A(q^{-1}) + B(q^{-1})P(q^{-1})]\, y(k) = P(q^{-1})B(q^{-1})y^*(k+1) \tag{3}$$

The coefficients of the L and P polynomials are obtained by solving

$$L(q^{-1})A(q^{-1}) + B(q^{-1})P(q^{-1}) = A^*(q^{-1})$$

(The roots of $A^*(q^{-1})$ are the desired closed-loop poles.) Matching coefficients of powers of q^{-1}, in this equation, results in a set of linear equations of the form $Mx = y$, where x and y contain the unknown coefficients and coefficients of A^*, respectively, and M is the eliminant matrix with a_i and b_i as its entries. It is well known that if the two polynomials $A(q^{-1})$ and $B(q^{-1})$ do not have any common roots, the eliminant matrix will be nonsingular. The inversion of this eliminant matrix causes severe problems, however, if the order of the system is overestimated (or if the estimated values for the parameters result in pole/zero cancellation). In such cases, the values obtained for the control law would become excessively large and could drive the system unstable.

In this Note, we propose to implement an order-variable control law, in which the order may change to reflect changes in the system or to correct inaccurate assumptions regarding the order of the system. To implement such a control law, parameter estimates for different model orders will be required. The identification algorithm used, which is also used to identify the appropriate order, is discussed in Sec. III. Finally, when zero steady-state error is desired, small modification of the previous equation results in integral action in the compensator. In all of the results to be presented, we have included integral action in the compensator. The major difference is that the eliminant matrix is of order $(2n + 1)$ with integral action amd $(2n - 1)$ without it (n is the assumed order of the plant). For brevity, we leave the details to Ref. 2.

III. Parameter Estimation and Order Determination

Lattice Filter

The least-squares lattice filter is used to estimate the parameters in Eq. (1) for all orders n up to and including a maximum order n_{max}. References 3 and 4 contain the derivation, complete algorithms, and many issues relevant to practical application of lattices. Because of space limitation, the algorithms (and the corresponding definitions) are not presented here. The residual error algorithm and the auto regressive (AR) coefficient algorithm used here are identical to the algorithms in Ref. 3. For the purpose of this Note, it suffices to note that $R_n^e(t)$ is the norm of the error vector resulted from a filter of order n.

Adaptive Order Determination

The (1,1) element of the positive definite matrix R_n^e is the square of the norm of an error that indicates the accuracy with which the model [Eq. (1)] fits the input/output data for the plant, for a given n. When the plant has the form of Eq. (1) for some order n, with no measurement noise, the first element of R_n^e decreases markedly, theoretically to zero, when the lattice order increases from $n - 1$ to n. Thus, the first element of R_n^e is an obvious candidate for the criterion for order determination. For the case where random noise is added to Eq. (1), various authors have devised methods for estimating the plant order.[5,6] These methods involve minimization of scalar functions of k, N, and fit-to-data criteria closely related to R_n^e. Generally, these methods require a great deal of time to select the appropriate order. Extensive simulations of the example in this paper, however, bear out the conclusion in Ref. 7 that a criterion for adaptively adjusting the order n in Eqs. (1–3) should involve the response of the control system as well as the plant order identified by the lattice filter. In this paper, two such criteria are used in conjunction, one taking precedence over the other.

Criterion 1

The criterion that takes precedence places bounds on the magnitude of the control $u(k)$. Two bounds on the magnitude are used, u_{sat} (for actuator saturation) and u_{max} (for controller order adjustment), with u_{max} greater than u_{sat}. The bound u_{sat} is used as a typical saturation limit; i.e., if the control law in Eq. (2), with order n, produces a $u_n(k)$ of magnitude greater than u_{sat}, the applied control at time k is set to $u_{sat} \cdot \mathrm{sign}[u(k)]$. The bound u_{max} is used as an indicator of order mismatch; i.e., if for this n, $|u_n(k)|$ is greater than u_{max}, then $u_{n-1}(k)$ and $u_{n+1}(k)$ are computed according to Eq. (2) with orders $n - 1$ and $n + 1$, respectively. The assumed order at time $k + 1$ is set to the integer j (either $n - 1$, n, or $n + 1$) such that

$$|u_j(k)| = \min[u_{n-1}(k),\ u_n(k),\ u_{n+1}(k)]$$

Criterion 2

If $|u_n(k)| \le u_{max}$, then the order-determination criterion similar to that in Ref. 7 is used:

$$e(k) = \frac{c}{\sum_{j=1}^{k} [y(j) - y^*(j)]^2}$$

where

$$\text{If } \log(R_n^e(1,1)) > \log(e(k)) + \Delta \Rightarrow n = n + 1$$
$$\text{If } \log(R_n^e(1,1)) < \log(e(k)) - \Delta \Rightarrow n = n - 1$$

If neither condition holds, the order is not altered. Constants c and Δ are chosen in accordance with the expected levels of noise, nonlinearities, etc. For the example in Sec. IV we have used $c = 0.0001$ and $\Delta = 1.5$. These values, obtained by trial and error, were used for several reference inputs in Ref. 8, with comparable results. For drastically different operating conditions, however, it is likely that different values would be needed.

Criterion 1 was motivated by the desire to avoid singularity of the eliminant matrix that must be inverted to solve for the coefficients in the polynomials L and P. We originally used the determinant of the eliminant matrix as a criterion for changing the order of the plant model; however, this was too sensitive and produced too frequent changes in the estimated plant order. The final form used in criterion 1 is related to the numerical condition of the eliminant matrix because, as the matrix becomes nearly singular, its inverse produces large control gains. The motivation for criterion 2 should be clearer. If the output approaches the desired output quickly, then the original assumed order need not be increased and may be decreased to try a smaller order. If after a large number of steps the error between the output and the desired output remains large, then $e(k)$ becomes small, forcing an increase in N unless there is virtually no prediction error. As results presented in the following section indicate, these criteria can be used to improve drastically the response of the system, though additional criteria may be developed to improve (and generalize) this approach further. In particular, note that criterion 1 is motivated by the pole-placement algorithm. For other control algorithms, different criteria may become necessary.

IV. Example

The mass-spring-damper system in Fig. 1 is to be controlled by an actuator that applies a force to the second mass. The measurement of the absolute velocity of the second mass is the single output. The physical parameters of the plant are as

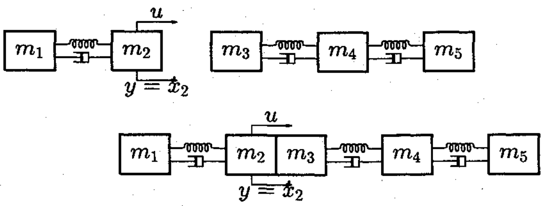

Fig. 1 System used for the example.

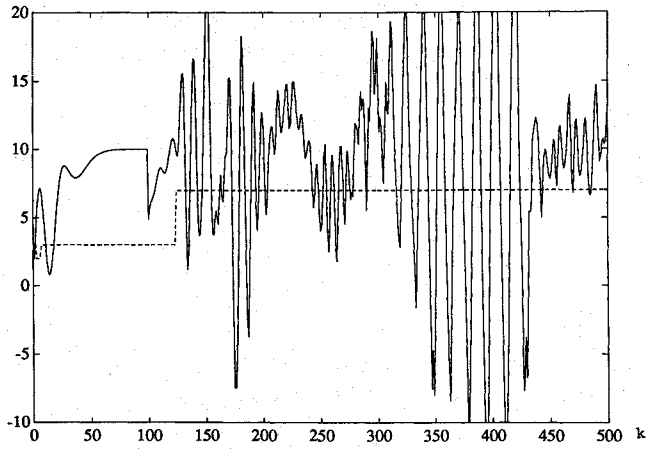

Fig. 2 Output: fixed-order controller ($n = 7$).

follows; $m_1 = 1$, $m_2 = m_3 = m_4 = 0.25$, $k_1 = k_3 = 8$, $k_2 = 16$, $c_1 = c_2 = 0.01$, and $c_3 = 0.001$. The length of the sampling interval is 0.05. (All constants are dimensionless.) For a simulation model, the discrete-time state-space representation was generated from the linear differential equations representing the plant.

It has been shown that, for fixed-order systems, the indirect adaptive pole-placement control in Eqs. (1-3) will converge asymptotically if the reference signal [y^* in Eq. (2)] is sufficiently exciting.[2,9] For a flexible system with several vibrational modes, however, following a reference signal that satisfies the excitation requirement might not be practical. In Figs. 2-4, the reference signal is constant [i.e., $y^*(t) \equiv 10$]. Although this signal is not sufficiently exciting, it is of the form most practical for applications. During the first 100 steps (sampling intervals), the plant consists only of the first two masses. After 100 samples, the second mass collides with and sticks to the third mass, producing a new plant consisting of four masses connected by springs and dampers. The state-space representation of the plant has order 4 before the impact and order 8 after the impact. However, because there is a rigid-body mode and only velocity measurement, the input/output model in Eq. (1) has order 3 before the impact and order 7 after the impact.

Simulation results are shown in Figs. 2-4. The solid line is the output (i.e., velocity of the second mass) and the dashed line is the estimated order of the plant. The system starts at rest with no deflection in the springs. The first 10 steps are used as a learning period. During this period, the control is set to the constant value 10. Criterion 2 in Sec. III is used alone to determine the order of the plant. In simulations, this criterion correctly selects the plant order (i.e., 3) before the end of the 10 step learning period if the order assumed initially is 2, 3, or 4. Between 10 and 100 steps, the control law described in Sec. II and the two order-determination criteria described in Sec. III are used.

During the first 25 steps after the impact, the adaptive control law is disengaged and a constant-gain proportional feedback is used. After this learning period, the adaptive controller is used to control the system. Since the nominal order of the system is 7, it is natural for the assumed order to be set to 7. However, as shown in Fig. 2, this leads to undesirable performance. This is at least partly due to the fact that the reference signals are not sufficiently exciting and, consequently, the parameter estimates for a seventh-order system result in undesirable closed-loop behavior.

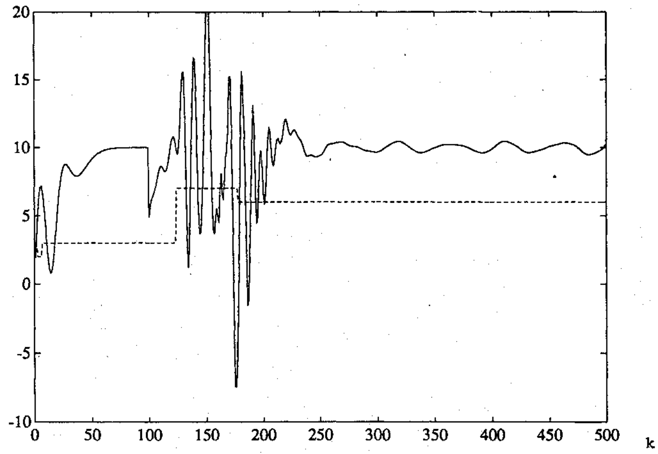

Fig. 3 Output: variable-order controller – initial order = 7.

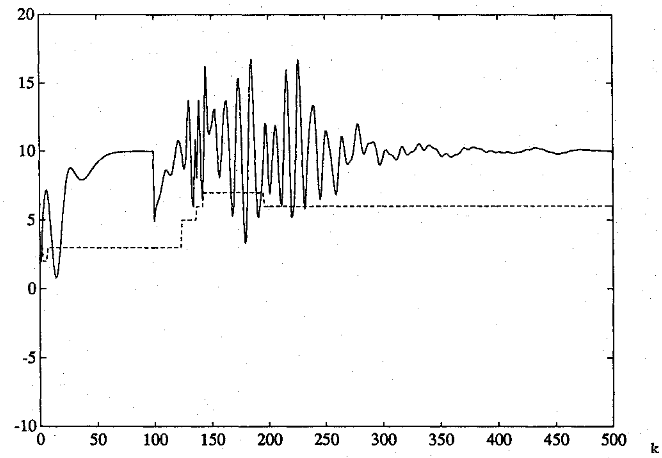

Fig. 4 Output: variable-order controller – initial order = 5.

As an alternative approach, we start the adaptive controller with the nominal plant order (i.e., 7), but allow the estimated value of the plant order to change according to criteria 1 and 2. The results that follow show the drastic improvement in the system behavior. Figure 3 shows the output of the system along with the estimated order of the plant (the dashed line). After a few steps (approximately 60), the order is reduced to 6 and stays at that value. Both cases had the same controller for the first 100 steps. The only difference between Figs. 2 and 3 is that, after the collision, in Fig. 3, the order of the plant is allowed to change, whereas in Fig. 2, the order is fixed (at 7) after the collision. It is interesting to note that, effectively, criterion 2 is not used after the collision because it dictates the highest allowable order ($N_{max} = 8$) for several hundred steps after the collision, and this order produces a control command that violates the constraint u_{max} in criterion 1, which takes precedence. This points to the inability of criterion 2, and similar criteria, to adjust quickly in the presence of sudden changes (such as collision), noise, and nonlinearities. The selected order, therefore, reflects the order that results in acceptable control law (rather than the estimated order for the plant). The only fixed-order that resulted in acceptable behavior was the case of fixed-order 6, which is not the actual order of the system and could not have been known a priori.

Finally, the following question arises: is 7 the best initial value for the order, and how is an appropriate order chosen if the nominal value is not known? The choice of an initial order should be made during the latter parts of the learning period (e.g., for k between 115 and 125). One plausible choice would be the order that minimizes

$$\left\{\sum_{k=115}^{125}|u_i(k)|,\ \forall i\right\}$$

where $u_i(k)$ is the control value obtained if the selected plant order were i. Notice that this period is the last 10 steps of the learning period, during which a proportional controller is used in real time. Once an order is chosen, the adaptive controller can be started. The motivation is to choose the order that results in the smallest amount of energy added to the system. The order that minimized

$$\left\{\sum_{k=115}^{125}|u_i(k)|\right\}$$

was 5. Figure 4 shows the system response when the order was set to 5 after the impact, but this order was allowed to vary (a fixed order at 5 would result in a plot similar to Fig. 2). The estimated value for the order is increased automatically from 5 to 7 and stays at 7 for approximately 50 steps, before returning to 6. The response obtained by this approach is at least as good as the response in Fig. 3, where the initial order of the controller was set to 7. Clearly, this provides only a partial answer to the previous question, in the sense that this approach was motivated, and works best, for this particular control law. We anticipate that more refined methods will result from further research. Nonetheless, it points to the importance of the choice of the control law in determination of the appropriate order for the adaptive controller.

A few issues concerning the implementation are noteworthy. Throughout the simulation, the lattice filter generates parameter estimates for all plant orders $N \le N_{max} = 8$. The lattice is not reinitialized after the impact, but the forgetting factor 0.99 is used to discard old data exponentially. The impact itself is detected by monitoring the position of the second mass. In all of the plots shown, we have used $u_{sat} = 35$ and $u_{max} = 100$. We have found that changing the value of the u_{sat} can result in different system behavior (e.g., the plots in Figs. 2-4 would be substantially different from $u_{sat} = 25$). The performance is much less sensitive to changes in u_{max}, however. In this example, u_{max} can vary from 70 to 130, whereas Fig. 2 stays exactly as shown here. For simplicity, real values were chosen for the desired poles (i.e., roots, of A^*). Complex poles, placed close to the real axis, would result in similar behavior. Generally, after the collision, placing the poles closer to the origin resulted in better system behavior. For the plots shown here, the roots of A^* are assigned to 0.75 and 0.2, before and after the collision, respectively.

V. Conclusions

In this paper, a variable-order adaptive controller has been much more effective than similar fixed-order adaptive controllers for control of a flexible system whose order changes abruptly during operation. Interestingly, the best order for the controller after the impact, in the example here, is one smaller than the true order of the plant. These results suggest that the criterion for adaptively adjusting the order should involve the computed control command, as well as a measure of fit to data for the desired model.

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Application of Encke's Method to Long Arc Orbit Determination Solutions

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Introduction

THE Laser Geodynamics Satellite (LAGEOS) was launched to perform geodynamics research in such areas as plate tectonic motion, long wavelength gravity field models, long period diurnal and semidiurnal tides, Earth rotation, and in related areas such as relativity.[1] To accomplish these studies, LAGEOS was constructed as a metal sphere approximately 60 cm in diameter with a mass of 407 kg and launched into a nearly circular orbit with an altitude of about 6000 km. The small area-to-mass ratio and high altitude reduce the effects of nongravitational forces. Embedded in its aluminum skin are 422 cube corner reflectors. LAGEOS is tracked by means of laser range measurements from a global set of ground stations, and to date, over 12 years of laser measurements have been collected. Long arc solutions of LAGEOS' orbit are performed specifically to study long period effects such as the time rates of change of the geopotential coefficients J_2 and J_3, the 18.6-year tidal effects, and other long period effects on the orbit.[2-4]

The numerical integration of the differential equations that model a satellite's orbital motion plays a significant role in the orbit determination process, particularly for precise solutions spanning thousands of orbital revolutions. When the batch filter is used to determine precise orbital solutions, it is affected by the numerical integration process in two ways. First, due to round off and truncation errors, the numerically inte-

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